

10. AN APPLICATION CASE OF THE PARATRIGONOMETRIC POLAR COORDINATES

10.1. Introduction

In the Chapter 6 we presented the basic elements of the Paratrigonometry (PRT). We remember that the fundamental equations of PRT are the following:

$$|spr_k \alpha|^k + |cpr_k \alpha|^k = 1 \quad (10.1)$$

$$tpr_k \alpha = tg \alpha \quad (10.2)$$

where $spr_k \alpha$ is "the paratrigonometric sinus of order k of the angle α ", $cpr_k \alpha$ is "the paratrigonometric cosinus of order k of the angle α " and $tpr_k \alpha$ is "the paratrigonometric tangent of order k of the angle α ".

Relation (10.2) represents the connection key between the Paratrigonometry (PRT) and the Classical Trigonometry (CT). As a matter of fact, this well known CT represents a particular case of the PRT, which is characterized by $k = 2$. Accepting PRT, the trigonometric functions in CT, $\sin \alpha$, $\cos \alpha$, $tg \alpha$ etc., represent the paratrigonometric functions of order 2 of the angle α . Another particular case of PRT is the Quadratic Trigonometry (QT) where $k = 1$. The bases of QT were given by Valeriu Alaci, professor of the University "Politehnica" of Timisoara from Romania [1].

From the relations (10.1) and (10.2) we can calculate the functions $spr_k \alpha$ and $cpr_k \alpha$, for any value of the "order" ($0 \leq k \leq \infty$), as a function of $tg \alpha$. Thus, for example:

$$spr_k \alpha = \pm |tg \alpha| / \left(1 + |tg \alpha|^k\right)^{1/k}. \quad (10.3)$$

The sign + (plus) or - (minus) is given to the function $spr_k \alpha$ by the known rule in CT, that is in function of the trigonometric quadrant where the angle α is situated.

The paratrigonometric functions are analyzed with the reference to the Cartesian coordinate system. In this system, the most frequently used in Mathematics, in PRT case the variable - angle α - is horizontally represented on the abscissa axis and the functions $spr_k \alpha$, $cpr_k \alpha$ etc. are represented vertically on the ordinate axis.