11. PARATRIGONOMETRIC FUNCTIONS RAISED TO SOME POWERS AND THEIR APPLICATIONS

11.1. Introduction

In Chapter 6 the authors showed that the basic functions "Paratrigonometric sinus of α", denoted $spr_k\,\alpha$, has various values and consequently various graphical forms depending of the "order" value k. In Figure 11.1 is represented the function $spr_k\,\alpha$ for various k values such as: $k = 0.25$, $k = 0.5$, $k = 1.0$, $k = 2.0$, $k = 4.0$ and $k = \infty$.

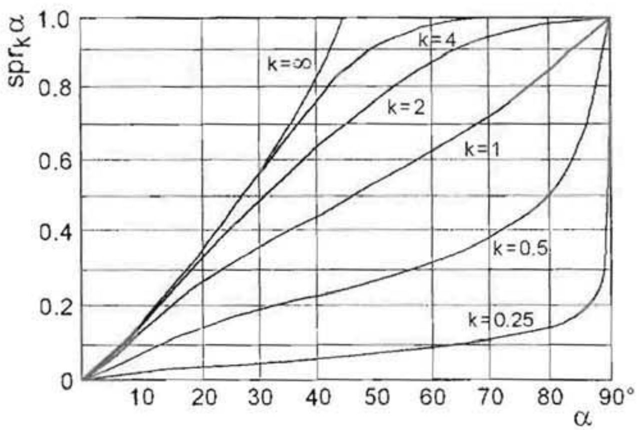

Fig. 11.1. The paratrigonometric function $spr_k\,\alpha$ for 6 values of the "order" k.

We recall that the fundamental relation in the Paratrigonometry is the following:

$$|spr_k\alpha|^k + |cpr_k\alpha|^k = 1 . \qquad (11.1)$$

Thus when $k = 2$, the relation (11.1) becomes the very well known relation of the Classical Trigonometry (CT) as such:

$$(\sin\alpha)^2 + (\cos\alpha)^2 = 1 . \qquad (11.2)$$

The relations (11.1) and (11.2) indicate the fact that CT represents a particular case of the Paratrigonometry (PRT), when $k = 2$.

The "key" connection between PRT and CT is the following relation:

$$tpr_k\,\alpha = tg\,\alpha\,. \tag{11.3}$$

This relation is valid for any value of *k*.

From the relations (11.1) and (11.3) we can obtain the following relation for $spr_k\,\alpha$ which can be calculated from the function $tg\,\alpha$, very well known from the CT:

$$spr_k\,\alpha = tg\,\alpha \,/ \left[1 + (tg\,\alpha)^k\right]^{1/k}. \tag{11.4}$$

Evidently that all the other paratrigonometric functions (cosine paratrigonometric etc.) can be calculated using the relations (11.1) and (11.4).

Returning to the Figure 11.1, we can see that in the spaces between the traced curves we can insert an infinite number of curves developed between the points with the coordinates (0°; 0) and (90°; 1.0), corresponding to the other values of *k* in the domain $0 \le k \le \infty$.

All these curves "fill in" the space limited by $O\alpha$ axis, the vertical line $\alpha = 90^\circ$ and the curve which represent the function $spr_\infty\,\alpha$. They all lock as a "bunch of fibers" which start from a single point and terminate in the other, but each fiber has the form dictated by the corresponding $spr_k\,\alpha$ function respectively. We mention that the segment $O\alpha$, between the lines $\alpha = 0^\circ$ and $\alpha = 90^\circ$, together with the segment on the vertical line $\alpha = 90^\circ$, between the limits $spr_k\,\alpha = 0$ and $spr_k\,\alpha = 1.0$, represents $spr_k\,\alpha$ for $k = 0$, as we have shown in Chapter 4.

In order to represent some periodic functions with a sinusoidal like form, which we meet very often in Physics and Technology respectively, we consider that there is a need for other "traces" of the above mentioned "fibers". For this purpose we can apply to the function $spr_k\,\alpha$ raised to some powers, thing that raise very much the area of the paratrigonometric modeling of some periodic functions with sinusoidal forms. If we denote by p the power at which is raised the paratrigonometric function $spr_k\,\alpha$ we can write $(spr_k\,\alpha)^p$.

In Figure 11.2, as an example, we traced the curves which represent the function $(spr_k\,\alpha)^2$ and in Figure 3 we traced the curves which represent the function $(spr_k\,\alpha)^{0.5}$ for the same values of order *k*, as in Figure 11.1.

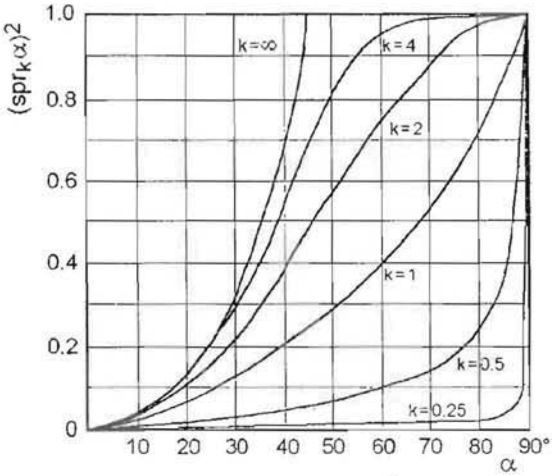

Fig. 11.2. The function $(spr_k\alpha)^2$ for the k values according the Figure 11.1.

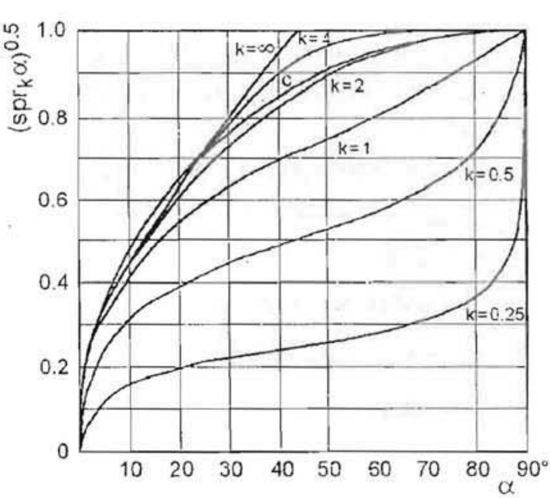

Fig. 11.3. The function $(spr_k\alpha)^{0.5}$ for the k values according the Figure 11.1.

Regarding the Figures 11.1, 11.2 and 11.3 we have to mention that by variation of k in the entire interval of possible values, such as $0 \le k \le \infty$ coupled with the variation of p in the entire interval of possible values, such as $0 \le p \le \infty$, we can cover the entire surface of the first trigonometric quadrant in the domain $0^\circ \le \alpha \le 90^\circ$ and $0 \le spr_k\alpha \le 1.0$. We must stress the fact that any similarity of a sinusoidal periodic function $F(\alpha)$ with the function $(spr_k\alpha)^p$ when k and p have constant values, can be accepted when $F(\alpha)$ exactly concide $(spr_k\alpha)^p$ definite by the values of k and p respectively. In Physics and Technology we can meet with many such cases when they coincide, but in many situations $F(\alpha)$ can not be