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MORE EXPLANATIONS ABOUT BAICA'S PROOF OF FERMAT'S LAST THEOREM

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ABSTRACT
In this paper the author will answer some questions raised at some various professional conferences and meetings when she presented her proof [1] of Fermat's Last Theorem.

KEY WORDS AND PHRASES

HASSE AND BERNSTEIN ALGORITHM (HBA)
JACOBI-PERRON ALGORITHM (JPA)
EULER-LAGRANGE THEOREM (ELT)
FERMAT'S LAST THEOREM (FLT)
BAICA'S GENERAL EUCLIDEAN ALGORITHM (BGEA)
EUCLIDEAN ALGORITHM (EA)
HILBERT'S UNIVERSAL ALGORITHM PERIODICITY PROBLEM (HUAPP)

ANSWERS TO THE QUESTIONS

QUESTION I

At the International Conference on Analytic Number Theory in Allerton Park, at the University of Illinois in Urbana-Champaign, May 16-20, 1995, the author was asked this question.

"Do you mean that if we come up with another new algorithm this time always periodic, than FLT is false?"

ANSWER TO QUESTION I

The answer is NO. In order for that new Algorithm to make FLT false there is a must for that new Algorithm to be The General Euclidean Algorithm.

This new General Euclidean Algorithm has to solve from its always periodicity. All of the problems in Chapter 2 of [1] as EA solves all of the problems in Chapter 1 of [1] from its always periodicity.

BGEA solves all of the problems in Chapter 2 of [1] up to its restricted periodicity. BGEA is the General Euclidean Algarithm since it is of the same cut or prototype as EA which is $n=2$ in BGEA, when Jacobi ($n=3$ in BGEA) and Perron (any n in BGEA) first originated it.

In Chapter 3 of [1] the author identified all her publications in which she partially proved up to BGEA restricted periodicity all of those open questions in n dimension (Chapter 2 of [1]).

In [1] the author proves explicitly HUAPP and with it she showed that BGEA is the General Euclidean Algorithm and this is the key in solving FLT as it is the key in solving completely now all of the problems in Chapter 2 of [1] from its restricted periodicity.

QUESTION II

At the Conference on Number Theory and Fermat's Last Theorem at Boston University on August 9-18, 1995, the author was asked this question.

"How do you relate the restricted periodicity of your BGEA with the degree n of the equation

$$x^n + y^n = z^n$$

in the FLT?"

ANSWER TO QUESTION II

Every $w=\sqrt{k}$ (quadratic irrational) makes EA always periodic. This is Euler-Lagrange if and only if Theorem. Therefore EA is periodic always and because of its always periodicity many problems in quadratics are solved (Chapter 1 in [1]) and the same problems in n dimensions were still open (Chapter 2 in [1]).

All of those open questions for $n>2$ caused Hilbert to ask for the invention of a universal algorithm of dimension n as powerful as EA for quadratics in order to solve all of the problems in higher ($n>2$) dimensions (Chapter 2 in [1]) from the periodicity of this universal Algorithm. Let's call this demand of Hilbert as HUAPP.

Since many great mathematicians before, including Hilbert, related quadratics with the periodicity of EA and since it is known that

$$x^2 + y^2 = z^2$$

has integral solutions is an immediate consequence of EA being always periodic, we can likewise perform the generalization for any n.

In [2], we proved that only some

$$w = \sqrt[n]{k} = \sqrt[n]{D^n + d} \text{ when } d \mid D$$

n-th degree irrationals makes BGEA periodic. This is Euler's direction in the proof of the periodicity of BGEA.

The Lagrange direction proof of the periodicity of BGEA is exactly the same as given by Perron [4,5], where he proved that his algorithm is periodic if w is an algebraic number or any n-th degree irrational ($Q=\sqrt[n]{k}$).

In [1] we proved that if $d \nmid D$, then BGEA is not periodic if $n \geq 3$, since otherwise BGEA will become Hilbert's Dreamed Universal Algorithm which will contradict HUAPP.

In conclusion the periodicity of EA was connected with the degree $n = 2$ of the irrational which makes EA always periodic and further with the existence of integral solutions

$$x^n + y^n = z^n \text{ when } n = 2.$$

Because some n-th degree irrationals make BGEA periodic the dimension of BGEA is n, and for $n = 2$, BGEA becomes EA, similarly we connect the dimension n of BGEA with the degree n in the equation

$$x^n + y^n = z^n$$

for $n \geq 3$, as the dimension $n = 2$ of EA in BGEA was connected with the dimension $n = 2$ in the equation

$$x^n + y^n = z^n,$$

by many other great mathematics in the History of Mathematics before.

QUESTION III

This question came in a letter received from Georg-August - Universitat in Göthingen, Germany dated 8-31-95. I received this comment:

"Finally let me comment on your paper. The crucial point, as it seems to me and also Prof. P., whom I asked, is that you want to generalize from quadratic fields and the Euclidean Algorithm to higher degrees and an extension of the algorithm.

But you never give a detailed proof how periodicity of the algorithm and solvability of Fermat's equation are connected. Therefore it remains very doubtful if such a connection exists.

In many cases, as for example the theory of complex functions of one or several variables, results and proofs cannot be generalized to "higher degrees" and even if there is an analogy of prerequisites and wanted results, it might be impossible to generalize the proof. This is the reason, why in our opinion your proof lacks completeness and must be worked out."

In anoter words this question can be rephrased as:

"What gives you permission to generalize from the quadratics and the always periodicity of EA to the n-dimension and the restricted periodicity of BGEA? "

This question is legitimate because the generalization may not be always possible.

ANSWER TO QUESTION III

This is correct. The BGEA is crucial. I argue that the Algorithm developed in the paper [2] and now called Baica's General Euclidean Algorithm (BGEA) is the tool that makes generalization possible.

The uses of a generalized algorithm and its periodicity were suggested by Hilbert.

What I have done is to extend the work of Hasse and Bernstein [3] to obtain tools that can be used to attack Fermat's Last Theorem.

What it was said is that it can be seen how the earlier paper that I wrote [2] and my paper on FLT [1] are connected.

The existence of a generalized algorithm BGEA allows an induction on the degrees of the Fermat's equations. In each case for $n \geq 3$, the algorithm BGEA is not periodic.

In Euclidean Variety, the principle of induction never fails to give the generalization.

The corresponding algebra of the Euclidean Geometry, in this case, is Elementary Number Theory, and that is a Peano-Algebra in which induction provides the generalization.

Fermat was thinking in the same direction to use induction for $n \geq 3$ but at that time he did not have the tool BGEA to be legitimate to use induction in his proof.

QUESTION IV

This question came in a letter received from Annali di Matematica Pura ed Applicata, Firenze, Italy on March 4, 1995.

Question:

"Let $w = \sqrt[n]{D^n + d}$ with D, d positive integers, $n \geq 3$ and d not a divisor of D.

It is possible to conclude that in such case BGEA is never periodic?"

If yes, how can this be proved? This proof was given in [1]. It is true that all the proofs for periodicity of HBA [5], and BGEA [2], considered $d \mid D$, and this is only a necessary condition to prove periodic.

In [1] the author proved explicitly HUAPP and putting those two together that will make $d \mid D$ an if and only if condition for the proof of the periodicity of BGEA.

All of the proofs in Chapter 2 of [1] from the restricted periodicity of BGEA are now if and only if conditions.

From logic the negation of an if and only if condition is again an if and only if condition and therefore it is true that if $d \nmid D$ then BGEA is not periodic.

In conclusion we solved more problems. Some of them include Hermite's Problem, Dirichlet's Problem, Hilbert's Problem and Galois' Theory of Polynomials Problem, not as controversial, but as difficult or more difficult than Fermat's Last Theorem using as the tool this restricted periodicity of the BGEA.

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