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## THE EUCLIDEAN CHARACTER OF THE FERMAT'S LAST THEOREM

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### ABSTRACT

In this paper the author is expressing her genuine concern about the proof of Fermat's Last Theorem in the Geometry of the Elliptic Curves or Elliptic Variety which may not be equivalent to the result in the Euclidean Geometry or Euclidean Variety, where Fermat's Last Theorem was initially originated, about three hundred fifty years ago.

### KEY WORDS AND PHRASES

EUCLIDEAN GEOMETRY (EG)  
GEOMETRY OF THE ELLIPTIC CURVES (GEC)  
EUCLIDEAN VARIETY (EV)  
VARIETY OF ELLIPTIC CURVES (VEC)  
FERMAT'S LAST THEOREM (FLT)

### INTRODUCTION

In Mathematics, problems can be approached geometrically, algebraically or analytically, and proofs can be given in different mathematical varieties, which are defined in the modern mathematics as mathematical models.

We can associate to each geometry a corresponding algebra and from here we can say that to every Geometric Variety it corresponds an Algebraic Variety, but the converse is not true.

In 1995 an in-house publication in the Annals of Mathematics at Princeton accepted a proof of Fermat's Last Theorem in the Geometry of Elliptic Curves and its corresponding Algebra, which was overwhelmingly embraced by the American Mathematical Society.

Considering the Euclidean character of FLT we show why FLT should not be accepted to be proved using Elliptic Curves.

### 1. THE STATEMENT OF THE PROBLEM

In [1] the author proved FLT using the restricted periodicity of her algorithm Baica's General Euclidean Algorithm (BGEA). Before its publication it was sent

for revision to the Annals of Mathematics and on January 23, 1995 the answer was "there is no justification given for the claim that the algorithm, BGEA, has anything to do with Fermat's Last Theorem."

After this decision it was sent for revision to the "Journal für die Reine und Angewandte Mathematik (Crelles Journal) and in February 7, 1995 the answer was:

"You have presented the above mentioned manuscript for publication in the Crelle's Journal. I regret to say that the Board of Editors has decided not to accept your paper for publication."

On February 13, 1995 I received another letter:

"Your manuscript has been sent to me to deal with. I am afraid that I did not find your "proof" convincing in the least. The London Mathematical Society is not interested in it."

Again, before its publication at the Conference on Number Theory and Fermat's Last Theorem held at Boston University on August 9-18, 1995 the author presented in a twenty minute talk the paper with her proof of FLT.

This problem has been a very controversial problem for about 350 years, it generated many unprofessional arguments at the conference from the opposing group. The author was severely insulted by the organizing people because she expressed her concern about the proof of FLT in the GEC.

She feels, that considering the Euclidean Character of FLT, it has to be proved in the EG or EV) and the proof in the GEC or VEC may not be equivalent to the result in the EV.

The necessary transformation to show equivalence and the Galois' connection from Category Theory to show that they are exactly the same results in two distinct categories, has not, in her view, be provided. At the conference she was ignored, insulted, ordered not to say anything about the Geometries and isolated to talk about her proof of FLT.

## 2. THE JUSTIFICATION THAT THE RESTRICTED PERIODICITY OF BGEA HAS EVERYTHING TO DO WITH THE SOLUTION OF FLT

In Mathematics we can construct as many Geometries or Geometrical Models as we please. All that we need is to have the elements declared, to state the axioms and the definition and to have consistency in our logic. For example, in the EG the elements are points and the straight line.

For Parabolic Geometry the elements are points and the lines are parabolas, for Elliptical Geometry the elements are points and the lines are ellipses. We have to make distinction what is an element in a geometry and what is a definition in a geometry.

At the FLT Conference in Boston, after the one hour talk titled "The Geometry of the Elliptic Curves", the author asked the invited speaker:

"Sir, do you recognize that the Geometry of the Elliptic Curves which you were describing right now is not Euclidean?"

Starting that moment the author was told that:

"You do not know any Geometry, of course it is Euclidean, do you not know that the conic sections are euclidean, etc.?"

At this reaction of my very legitimate question, I was surprised that these very fine Algebraic Geometrists do not make the distinction that the elliptic curves are elements in the Geometry of Elliptic Curves, while they are definitions in the Euclidean Geometry.

Every Geometry has its corresponding associate Algebra. Only one geometry is the Euclidean Geometry (EG), the other geometries are non Euclidean Geometries.

The geometries do not report to each other, but they all report to the Topology. Because of this, if you prove something in one geometry it may not be the same as in other geometries.

Let's look for the example at the V-th postulate in EG, what is becoming of it in the Parabolic Geometry (PG)?

Something nice can happen when the results are equivalent. The necessary transformation from Elliptic to Euclidean is required to show that those results proved in two distinct geometries are equivalent.

But this is not enough once it is proved equivalent, in order to show that the results are the same there is a need to provide the Galois' connection from Category Theory to show that they are exactly the same results in those two distinct categories.

We all know that Hasse solved parametrically the diophantine equation of the form:

$$x^2 \pm y^2 = z^2.$$

Because this equation is a homogeneous equation in [2] the author found a transformation to transform Hasse's equation into Pell's equation and solved explicitly Hasse's equation.

This transformation is from Euclidean Variety into Euclidean Variety and it was possible only because Hasse's equation is homogeneous. Of the Elliptic Curves the required transformation (isomorphism) is from Elliptic to Euclidean and it seems to be more complicated and it may not exist.

In this case, the proof of FLT in the Geometry of Elliptic Curves will be much longer than it is now, and will exceed the margins a lot more.

The conjecture known as FLT was stated by Fermat on the margin of his copy of Bachet's translation of Diophantus, at the side of Problem 8 of Book 2:

"To divide a given square number into two squares."

Fermat's marginal note reads: "To divide a cube into two cubes, a fourth power, or in general any power whatever into two powers of the same denomination above the second is impossible, and I have assuredly found an admirable proof of this, but the margin is too narrow to contain it."

Whether Fermat really possessed a sound proof of this problem for some mathematicians, will probably forever remain an enigma. It is not an enigma for me.

At that time Fermat anticipated a proof by induction. It was not a legitimate induction without the Baica's General Euclidean Algorithm.

The induction has to be made on the dimension of BGEA considering its restricted periodicity for  $n \geq 3$ .

It is known that everything that is proved in quadratics from the periodicity of Euclidean Algorithm are if and only if results.

One immediate consequence of EA being periodic is that

$$x^2 + y^2 = z^2$$

has integral solutions, and this like all the other results mentioned in [1] is an if and only if result.

Likewise BGEA being the only General Euclidean Algorithm proves up to its restricted periodicity all those similar results in  $n$  dimensions, which like in quadratics, are if and only if results.

Therefore BGEA is always periodic if and only if Fermat is false, and this will bring us to BGEA not always periodic for  $n \geq 3$  if and only if FLT is true. It is an inductive generalization from quadratics.

The restricted periodicity of the BGEA gives now a complete solution to many other open problems in  $n$  dimensions.

This includes solutions for Hermite's problem Dirichlet's problem, Hilbert's problem and Galois' theory of polynomials problem, not as controversial but as difficult or more difficult than Fermat's Last Theorem.

In conclusion I do believe that those who misjudged me will realize that the restricted of BGEA have everything to do with the solution of FLT. Up to this time, [1] is the only proof of FLT in Euclidean, and considering the Euclidean character of FLT, the History of Mathematics and the role of the greatest mathematicians who led the author to the proof of FLT in [1] should not be undermined.

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*PERIODICITY.*