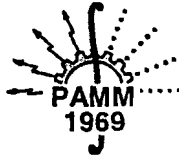


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Periodical of the

*Pannonian
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BAM-CIX/2006, Pg.2265-2270
Lecture at the PC-148/B.alm.

! Profs
A.Peretti & M.Baica
Argentina USA-Romania

An abridged method to ^{DERIVE THE} asymptotic
formula for the *Goldbach decomposition*.
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Caretaken by the *PAMM-Centre* at the

***Budapest University of Technology
and Economics (BUTE)***



1782

BUDAPEST

B.almádi

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ISSN 1417 278 X (series nr. for MB-volumes)

AN ABRIDGED METHOD TO DERIVE THE ASYMPTOTIC FORMULA FOR THE GOLDBACH DECOMPOSITIONS

by Aldo Peretti & Malvina Baica

ABSTRACT

The purpose of this paper is to give a shorter prove of the authors previous results on the Binary Goldbach's Problem [1].

0 INTRODUCTION

In ref.[1] the authors have given a method in order to obtain an exact formula for the Hardy-Littlewood function.

$$v(t) = \sum_{t=p_1+p_2} \log p_1 \cdot \log p_2$$

Here is indicated how the use of the tauberian theorem quoted in Lemma 3 of ref.[1] enables us to obtain a much shorter derivation of the asymptotic formula for $v(t)$.

1 THE STARTING FORMULA

As was proved in ref.[1], we have that

$$L(v(t)) = e^s \left(\frac{1-e^{-s}}{s} \right)^2 \left\{ \sum_q^N \sum_h \frac{\mu(q)}{\varphi(q)(s+2\pi i h/q)} + AN^{2\theta+1/2+\varepsilon} \right\}^2$$

where L denotes the Laplace transform, θ is the upper bound of the real part of the imaginary zeros of the L-series involved, and the formula is valid of $\theta \geq 3/4$, which is the actual case.

We write it as

$$(1.1) \quad \begin{aligned} L\{v(t)\} &= e^s \left(\frac{1-e^{-s}}{s} \right)^2 \left\{ g_N(s) + AN^{2\theta+1/2+\varepsilon} \right\} \\ &= e^s \left(\frac{1-e^{-s}}{s} \right)^2 \left\{ g_N^2(s) + 2g_N(s)AN^{2\theta+1/2+\varepsilon} + A^2N^{4\theta+1+2\varepsilon} \right\} \end{aligned}$$

Due to the fact that

$$L^{-1}\left\{e^s\left(\frac{1-e^{-s}}{s}\right)^2\right\} = \begin{cases} t & \text{if } 0 < t < 1 \\ 2-t & \text{if } 1 < t < 2 \\ 0 & \text{otherwise} \end{cases}$$

the term at the right hand side of (1.1) has not any relevance for $v(t)$, and we can put

$$L\{v(t)\} = e^s\left(\frac{1-e^{-s}}{s}\right)^2 \left\{g_N^2(s) + 2g_N(s)AN^{2\theta+1/2+\varepsilon}\right\}$$

if $t > 2$.

By the tauberian theorem of Lemma 3 of ref. [1]

$$(1.2) \quad v(t) \sim L^{-1}\left\{g_N^2(s) + 2AN^{2\theta+1/2+\varepsilon}g_N(s)\right\} =$$

$$= L^{-1}\left\{\sum\sum\frac{\mu^2(q)}{\varphi^2(q)(s+2\pi i h/q)} + \sum_{q_1 \neq q_2}\sum\sum\sum\frac{\mu(q_1)\mu(q_2)}{\varphi(q_1)\varphi(q_2)(s+2\pi i h_1/q_1)(s+2\pi i h_2/q_2)}\right.$$

$$\left. + 2AN^{2\theta+1/2+\varepsilon}\sum\sum\frac{\mu(q)}{\varphi(q)(s+2\pi i h/q)}\right\} =$$

$$= \sum_{q=1}^N\frac{\mu^2(q)}{\varphi^2(q)}C_q(t) \cdot t + \sum\sum\sum\sum\frac{\mu(q_1)\mu(q_2)e^{-A_1t} - e^{-A_2t}}{\varphi(q_1)\varphi(q_2)A_2 - A_1} +$$

$$+ 2AN^{2\theta+1/2+\varepsilon}\sum_{q=1}^N\frac{\mu(q)}{\varphi(q)}C_q(t)$$

But

$$\sum_{q=1}^N\frac{\mu^2(q)}{\varphi^2(q)}C_q(t) \cdot t = \sum_{q=1}^{\infty}\frac{\mu^2(q)}{\varphi^2(q)}C_q(t) \cdot t + \delta_1 e^{3\gamma d(t)}\frac{(\log \log N)^2}{N}\log \log t \cdot t$$

$$(1.3) \quad \left|\sum\sum\sum\sum\frac{\mu(q_1)\mu(q_2)e^{-A_1t} - e^{-A_2t}}{\varphi(q_1)\varphi(q_2)A_2 - A_1}\right| \leq \frac{\delta_2}{2\pi}N^2(N+1)^2$$

$$\vartheta(t) - \vartheta(t-1) \sim \sum_{q=1}^N\frac{\mu(q)}{\varphi(q)}C_q(t)$$

(Where $\vartheta(t)$ is the Chebishev function $\vartheta(t) = \sum_{p \leq t} \log p$)

as was shown in ref.[1] by Lemmas 1, 2, 3.

Hence

$$(1.4) \quad v(t) \sim \sum_1^{\infty} \frac{\mu^2(q)}{\varphi^2(q)} C_q(t) \cdot t + \delta_1 t e^{3\gamma} d(t) \frac{(\log \log N)^2}{N} \log \log t$$

$$+ \frac{\delta_2}{2\pi} N^2 (N+1)^2 + \delta_3 2AN^{2\vartheta+1/2+\varepsilon} \log t$$

We choose now $t = N^5$, so that

$$(1.5) \quad v(t) \sim \sum_1^{\infty} \frac{\mu^2(q)}{\varphi^2(q)} C_q(t) \cdot t + \delta_1 e^{3\gamma} d(t) (\log \log t)^3 \cdot t^{4/5}$$

$$+ \frac{\delta_2}{2\pi} t^{4/5} + \delta_3 2A t^{5 \cdot \frac{2}{5} \vartheta + \frac{1}{10} + \varepsilon} \log t$$

It is evident now the little influence that the value of ϑ has upon the value of $v(t)$.

The preceding formula coincides, in its essential features, with that deduced by the exact method.

Due to the multiplicative properties of $\mu(q)$, $\varphi(q)$ and $C_q(t)$ the series in the first term at right can be written as:

$$(1.6) \quad \sum_1^{\infty} \frac{\mu^2(q)}{\varphi^2(q)} C_q(t) = 2 \prod_{p=3}^{\infty} \left(1 - \frac{1}{(p-1)^2} \right) \prod_{p|t} \frac{p-1}{p-2}$$

$$= 1,3203 \prod_{p|t} \frac{p-1}{p-2}$$

So that (1.5) turns out to be

$$v(t) \sim 1,3203 \prod_{p|t} \frac{p-1}{p-2} \cdot t + O(t^{\frac{4}{5}+\varepsilon})$$

From (1.5) follows, as was shown in ref.[1], that the Goldbach hypothesis is correct for even $t > 10^{60}$.

REFERENCES

- [1] Baica, M.- Peretti, A., The Binary Goldbach Problem-www.peretti.da.ru