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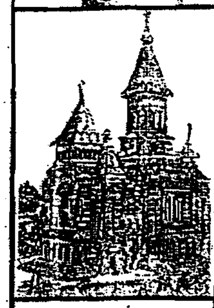


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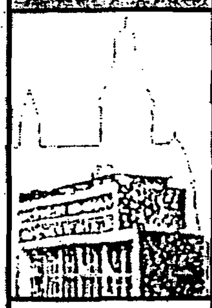

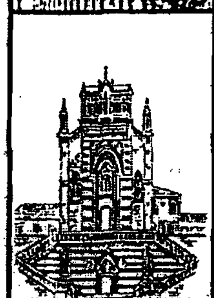


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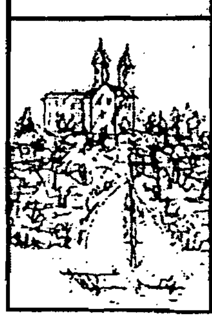


A GENERALIZED DESCARTES (GOLDBACH) BINARY PROBLEM

By AldoPeretti and Malvina Baica***

Abstract. In this paper the author will introduce a Diophantine Equation which can be considered as generalized form of Descartes (Misnamed Goldbach) binary problem.

The authors do not solve the problem but give some important hints which could lead to the solution of this problem.

Keywords and phrases: Goldbach binary problem, Diophantine Equation

1. Introduction

In this paper we deal with the quantity of solutions of the Diophantine equation

(1.1) $$t=p_1+p_2^k$$

where $k >> 2$, t is an even natural number, and p_1, p_2 are odd primes.

The case k=1 is of course the classical binary Goldbach problem.

Needless to say, it is commonly considered as a more intractable problem that was the Goldbach problem in its moment.

We do not solve this problem, but establish the firm grounds on which it could be solved.

At difference with the binary Goldbach decomposition not every small even number is decomposable in the form (1.1), and it seems at first sight that it is highly doubtful if the majority of even numbers admit such decomposition.

2. The starting formula

Denote with N (t) the quantity of solutions of equation (1.1) then we have the exact relation

(2.1) $$N(t)=\int_0^t \Delta\pi(x+1)\Delta\pi_k(t-x).dx$$

deduced in ref. [1], where

$$\Delta\pi(x)=\pi(x)-\pi(x-1) \qquad \Delta\pi_k(x)=\pi\left(\sqrt[k]{x}\right)-\pi\left(\sqrt[k]{x-1}\right)$$

Furthermore, we have the asymptotic relations

(2.2) $$\Delta\pi(x)\sim\sum_{q=1}^{\infty}\frac{\mu(q)}{\varphi(q)}\frac{C_q(u)}{\log u}$$

deduced in ref. [1], and

(2.3) $$\Delta\pi_k(x)\sim\frac{x^{1/k-1}}{\log x}\sum_{q=1}^{\infty}\sum_{\substack{h=0\\(h,q)=1}}^{q-1}\frac{W(k,q,h)}{\varphi(q)}e^{-2\pi i\frac{h}{q}x}$$

Replacing the appropriate values of (2.2) and (2.3) in (2.1) we get:

$$N(t)\square\int_0^t\left\{\sum_{q_1=1}\frac{\mu(q_1)}{\varphi(q_1)}\frac{C_{q_1}(x+1)}{\log(x+1)}\right\}\left\{\frac{(t-x)^{1/k-1}}{\log(t-x)}\sum_{q_2=1}^{\infty}\sum_{h}\frac{W(k,q_2,h)}{\varphi(q_2)}e^{-2\pi i\frac{h}{q_2}(t-x)}\right\}.dx$$

(2.4)

$$=\int_0^t\sum_{q_1=1}^{\infty}\frac{\mu(q)}{\varphi(q)}\frac{C_q(x+1)}{\log(x+1)}\frac{(t-x)^{1/k-1}}{\log(t-x)}dx+$$

$$+\int_0^t\left\{\sum_{q=1}^{\infty}\frac{\mu(q)}{\varphi(q)}\frac{C_q(x+1)}{\log(x+1)}\right\}\left\{\frac{(t-x)^{1/k-1}}{\log(t-x)}\sum_{q=2}^{\infty}\sum_{h}\frac{W(k,q,h)}{\varphi(q)}e^{-2\pi i\frac{h}{q}(t-x)}\right\}dx$$

3. **The singular series of the problem**

According to what occurs in the binary Goldbach problem, we know that the dominant term in (2.4) is obtained when we select there only the terms with $q_1 = q_2$ Hence we can put

(3.1) $$N(t)\square\int_0^t\sum\frac{\mu(q)}{\varphi^2(q)}C_q(x)\left(\sum W(k,q,h)e^{-2\pi i\frac{h}{q}(t-x)}\right)\frac{(t-x)^{1/k-1}\,dx}{\log(x+1).\log(t-x)}$$

Performing integration by parts we obtain:

(3.2)
$$N(t)\square\sum\frac{\mu(q)}{\varphi^2(q)}C_q(t)\sum W(k,q,h)\int_2^t\frac{(t-x)^{1/k-1}\,dx}{\log(x+1).\log(t-x)}+T_0$$

$$\square\sum\frac{\mu(q)}{\varphi^2(q)}C_q(t)\sum W(k,q,h)\frac{k\,t^{1/k}}{\log^2 t}+T_0$$

where T_0 is the second term of the integration by parts.

If we assume that T_0 is a negligible term with respect to the first one, then the series in (3.2) is the singular series to the problem.

If in change we apply the first mean value theorem of the integral calculus:

$$\int_p^q f(x)g(x).dx=f(\xi)\int_p^q g(x).dx \qquad (p\le\xi\le q)$$

to (3.1) we obtain:

$$N(t) \approx \sum_q \frac{\mu(q)}{\varphi^2(q)} C_q(\xi) \left(\sum_h W(k,q,h) e^{-2\pi i \frac{h}{q}(t-\xi)} \right) \int_0^t \frac{(t-x)^{1/k-1}\, dx}{\log(x+1)\log(t-x)}$$

(3.3)

$$\approx \sum_q \frac{\mu(q)}{\varphi^2(q)} C_q(\xi) \left(\sum_h W(k,q,h) e^{-2\pi i \frac{h}{q}(t-\xi)} \right) \frac{k\, t^{1/k}}{\log^2 t}$$

From (3.3) follows the theorem:
The equation (1.1), for arbitrary fixed k, has solution only for those values of n such that the singular series is positive.

4. Comparison with known results

L. K. Hua has proved, by the circle method, in ref [1] that the equation has solution for almost all even numbers.

Reference

[1] L. K. Hua: Some results in the additive prime number theory. Quart J. Math Oxford 9(1938) p. 68 - 80

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