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AN APPLICATION CASE OF THE PARATRIGONOMETRIC POLAR COORDINATES

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Abstract. In this paper we apply the polar coordinates to define a complete denture of a toothed wheel mathematically, using the paratrigonometric function "paratrigonometric sinus" for four values of paratrigonometric order k.

Keywords and phrases: Paratrigonometry, Polar coordinates, Toothed wheels.

1. Introduction

In [3] we presented the basic elements of the Paratrigonometry (PRT). We remember that the fundamental equations of PRT are the following:

$$|spr_k\alpha|^k + |cpr_k\alpha|^k = 1 \qquad (1.1)$$

$$tpr_k\alpha = tg\alpha \qquad (1.2)$$

where $spr_k\alpha$ is "the paratrigonometric sinus of order k of the angle α", $cpr_k\alpha$ is "the paratrigonometric cosine of order k of the angle α" and $tpr_k\alpha$ is "the paratrigonometric tangent of order k of the angle α".

Relation (1.2) represents the connection key between the Paratrigonometry (PRT) and the Classical Trigonometry (CT). As a matter of fact, this well known CT represents a particular case of the PRT, which is characterized by $k = 2$ [3]. Accepting PRT, the trigonometric functions in CT, $sin\alpha$, $cos\alpha$, $tg\alpha$ etc, represent the paratrigonometric functions of order 2 of the angle α. Another particular case of PRT is the Quadratic Trigonometry (QT) where $k = 1$. The bases of QT were given by Valeriu Alaci, professor of the "Politehnica" University from Timişoara, Romania [1].

From the relations (1.1) and (1.2) we can calculate the functions $spr_k\alpha$ and $cpr_k\alpha$, for any value of the "order" ($0 \leq k \leq \infty$), as a function of $tg\alpha$. Thus, for example:

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$$spr_k\alpha = \pm\, |tg\alpha| / (1+|tg\alpha|^k)^{1/k} \qquad (1.3)$$

The sign +(plus) or –(minus) is given to the function $spr_k\alpha$ by the known rule in CT, that is in function of the trigonometric quadrant where the angle α is situated.

The paratrigonometric functions are analyzed with the reference to the Cartesian coordinate system. In this system, the most frequently used in Mathematics, in PRT case the variable – angle α – is horizontally represented on the abscissa axis and the functions $spr_k\alpha$, $cpr_k\alpha$ etc are represented vertically on the ordinate axis.

In addition to the paratrigonometric functions an important roles in PRT have also the "Basic Trigonometric Figures" (BTF). Generally, these represent what the "trigonometric circle" (with radius $R = 1$) is in CT and respectively the "trigonometric rhombus" (where all angles are right angles) in QT.

In [3] we determined that the BTF equation is:

$$|y|^k + |x|^k = 1 \qquad (1.4)$$

where k is the "order" of the paratrigonometric function to which is the respective BTF referring – see relations (1.1) and (1.2) - . Thus, as an example, for the trigonometric circle, characteristic for CT, $k = 2$ and for the trigonometric rhombus, characteristic for QT, $k = 1$.

As in a normal way, the paratrigonometric functions are represented in the Cartesian coordinate system, also BTFs are represented in these coordinates.

In Figure 1 we represent, in the Cartesian coordinate system with the abscissa Ox and ordinate Oy, all the BTFs for $k = 1$, $k = 2$, $k = 4$ and $k = \infty$ in all four quadrants (I – IV).

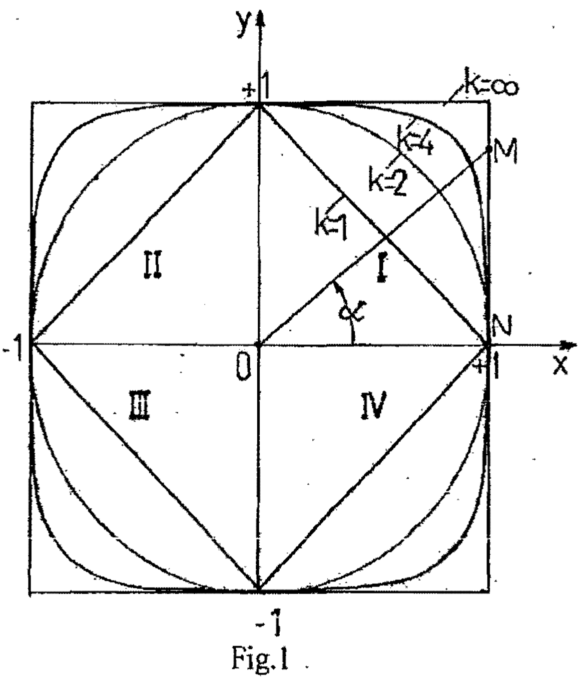

Fig.1

It is interesting to remark the closeness between our results in regard to the BTF, totally independent obtained in comparison with the "quadrilobes", ample analyzed with a very developed and sophisticated mathematical tool by the professor Mircea Eugen Şelariu of "Politehnica" University of Timişoara, Romania [6].

We consider that the Paratrigonometry can have multiple technical applications in engineering. On the other way, in the technique we see many processes where rotation motion intervenes and these, in there turn, can be very well mathematically modeled using these polar coordinates.

In next chapter we will analyze one of these cases where the polar coordinates in PRT are applied.

2. The paratrigonometric functions represented in polar coordinates, applied in the technology of toothed wheels

The toothed wheels are very often used in the domain of the machines and mechanical installations in the diverse transmission systems with chain, conveyers, elevators, the rolling systems with caterpillars and especially gearings [4]. In function of their utilization the teethes profile have diverse geometrical forms; this problem does not constitute the subject of this paper.

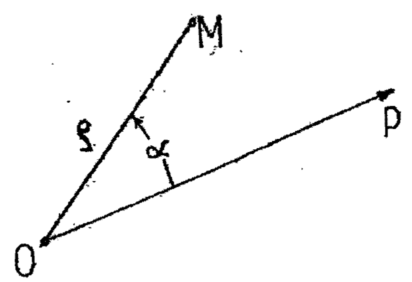

Fig.2

The characteristic elements of a system in polar coordinates (see Figure 2) [5] are the following:

- The pole O which constitutes the origin point for polar axis Op and for polar radius ρ;

- The polar axis Op in function of which the polar angle value α is measured;

- The polar radius ρ measured from the origin O to the point M whose position is determined by the polar coordinates α and ρ;

- The polar angle α.

The position of the point M for which a specific value of the polar angle α is well determined if the polar radius ρ given by the function $\rho = \rho(\alpha)$ is known. The function

ρ = ct. represent the mathematical model of a circle with the center in O having the radius

$R = \rho$. Thus, for example, the trigonometric circle in QT is represented by the function

$\rho = 1$.

Returning to the toothed wheels we remember that actually the designing and their machining is performed "tooth by tooth". The periphery of the toothed wheel is performed by placing on it of an integer number of the successive pairs tooth-gap.

In what is following we propose to establish the mathematical model for the entire denture of toothed wheel. If we use unit measurements for the angles this denture is developing on that angle which characterize a whole circle, namely $(2.\pi)$ rad. or 360^0. It is critical that, in this given situation, is convenient or almost mandatory to use polar coordinates. Since we will work with very small angles we choose as measurement units the "degree" and not the "radian". We consider that the number of the teeth composing the denture of the toothed wheel is z. They are disposed on the circle with the average polar radius ρ_m, as it is represented in Figure 3. The total height of the tooth is $(2.h)$. The head of the tooth with the height h is its portion between the average radius ρ_m and exterior radius ρ_e. The foot of the tooth, having the same height h, is its portion between the interior radius ρ_i and ρ_m. The polar radiuses mentioned as well as the current radius ρ, they all have the origin in the pole O, which coincide with the center of the toothed wheel. For a total number of teeth (in fact pairs tooth-gap) z means that the angle which corresponds to a such pair is $\alpha_z = (360/z)^0$. If we desire to express mathematically the contour of such pair by the periodic function $spr_K\varphi$, for example, means that it has to reproduce a complete variation for a period of 360^0 in the course of the angle α_z only. This condition is fulfilled when the variable φ of the function $spr_K\varphi$ is given by the product $\varphi = z.\alpha$, where φ is the polar angle. In this way, with α_z we have $\varphi = 360^0$. Thus the complete period for the $spr_K\varphi = spr_k(z.\alpha)$ is α_z. During of this period the function $spr_k(z.\alpha)$ varies having successive values 0 and respectively +1 and -1.

The order of the function, represented by k, is that one which is dictated in our case by the tooth profile form. For simplification, we accept $h = 1$. In this case, considering what we have shown above, the polar radius ρ for the points which form the denture profile of a toothed wheel is given by the relation:

$$\rho = \rho_m + spr_k(z.\alpha) \tag{2.1}$$

Based on this relation, in the Figures 3...6 we represented the contours of three teeth, for $\rho_m = 6.h$ and for the four values of k for which we represented BTFs in Figure1.
Thus, in Figure 3 we represented the case $k = 1$, in Figure 4 the case $k = 2$, in Figure 5 the case $k = 4$ and in Figure 6 we refer to the case $k = \infty$.

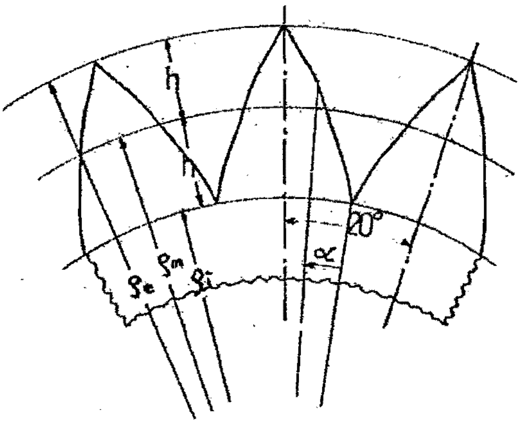

Fig.3

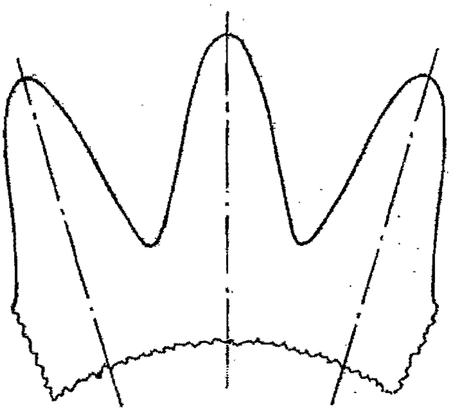
Fig.4

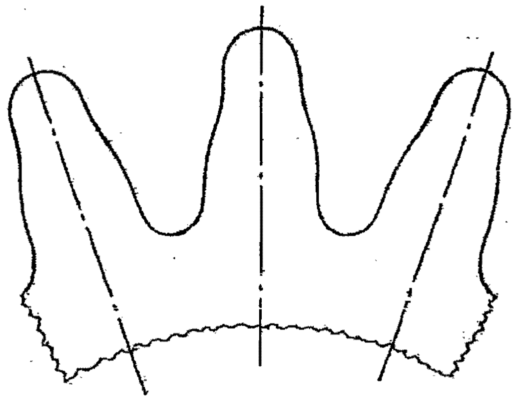
Fig.5

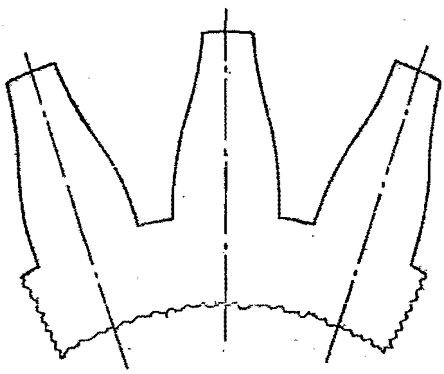
Fig.6

Regarding the situation when $k = \infty$ (Fig.6) we like to mention that under an algebraic aspect it is about to solve some problems in a "limit" case as we proceeded in a previous paper [2]. For a better understanding of the function $spr_\infty\alpha$ variation we can appeal to the BTF representation for $k = \infty$ of Figure 1. We see that in the first trigonometric quadrant, in the domain $0 \leq \alpha \leq 45^0$, $spr_\infty\alpha$ is represented by the line segment MN. On the other side $tg\alpha$ = MN/ON and since ON = 1, we will have $spr_\infty\alpha = tg\alpha$. For the domain $45^0 \leq \alpha \leq 90^0$ we will have $spr_\infty\alpha = 1$, as we can see in Figure 1.

For the other trigonometric quadrants (II...IV) we will have $|spr_\infty\alpha| = |tg\alpha|$, for the domains $135^0 \leq \alpha \leq 180^0$; $180^0 \leq \alpha \leq 225^0$ and $315^0 \leq \alpha \leq 360^0$. For the other domains of the angle α, we will have $spr_\infty\alpha = \pm 1$, as we can see in Figure 1. The signs + (plus)
or – (minus) should be applied, in all cases, according the known rule from CT.

Of course, for our case represented in Figure 6, everything what we have shown above regarding the domains of the values for α are valid regarding the angle $\varphi = (z.\alpha)$.

3. Conclusions

Considering what we have analyzed in the previous chapters we have the following important conclusions:

3.1. In using the Paratrigonometry for applications there are situations when it is recommended to use the polar coordinates instead of the Cartesian coordinates.

3.2. A situation for which the use of the polar coordinates is preferable is the paratrigonometric modeling of the toothed wheels dentures.

This modeling permit a unique relation to represent of the entire denture of the toothed wheels, in contrast with the present situation when their designing and machining is performed using the method "tooth with tooth". The relation which represents the entire denture of the toothed wheel contains the average polar radius of this denture and the paratrigonometric function "paratrigonometric sinus of order k" of the product between the current polar angle and the total number of the teeth.

3.3. The teeth profiles is a function of the value of the paratrigonometric order k.

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The list of the figures

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