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Malvina BAICA

Abstract

A. Wiles announced his proof of the elliptical Fermat's Last Theorem (ELFLT) in the summer of 1993. At that time the author was concerned that this (ELFLT) is not the same as Fermat's Last Theorem in Euclidean terms (EFLT) where Fermat's Last Theorem (FLT) originated about three hundred fifty years ago. Also, the author was concerned about using low levels when the higher levels of the Euler Systems were not constructed in the number theory of the elliptic curves.

In this paper the author will show that G. Faltings finished the proof of (ELFLT) using deformations instead of higher levels of the Euler Systems on March 1995. Also, the author used the Baica's Generalized Euclidean Algorithm (BGEA) as the only Euler System of the Algebraic Number Theory and proved (EFLT) on April 1994. We will show that (ELFLT) and (EFLT) are equivalent but not the same, proving that only an isomorphism can be found and the functor to show that they are the same can not be produced.

Key words. Fermat's Last Theorem, Euclidean Fermat's Last Theorem, Elliptical Fermat's Last Theorem

2000 AMS Subject Classification Codes:10B10, 10B15, 10A30, 12A45

1 Introduction

Helmut Hasse, who was the author's PhD Dissertation Advisor, once stated: "The end of the 20^{th} century will bring the solution for Fermat's Last Theorem (FLT) and the solution will come from the Euclidean Number Theory, Algebraic Number Theory tools, as Fermat had intended".

K. F. Gauss invented the Algebraic Number Theory to be the algebra of the n-Dimensional Euclidean Geometry (E^nG) and therefore it has to be proved in the algebraic number theory. Gauss, who is called the prince of mathematicians, said that he was not going to waste his professional life in attempting to prove (FLT), which he may not have been able to prove anyway, and that he would leave the problem to better mathematicians to solve it.

A. Wiles [16] announced his proof of the (ELFLT) on August 1993.

On May 1994, the author [2] presented her algorithmic Euclidean proof of (EFLT) at the International Conference on Analytic Number Theory in Allerton Park, at the University of Illinois in Urbana - Champaign. A. Wiles's proof was declared finally completed on September 1994. In 1995, the *Annals of Mathematics*, a publication at Princeton [15, 16], accepted a proof of (FLT) in the geometry of elliptic curves and its corresponding number theory, which is Hecke algebra.

The July 1995 *Notices of the AMS* published a translation from the German of G. Faltings' March 1995 article [11], in which the author said at the beginning of the paper: "The proof of the conjecture mentioned in the title was finally completed in September of 1994. A. Wiles announced this result in the summer of 1993. However, there was a gap in his work: the paper of Taylor and Wiles ([15]) does not close this gap but circumvents it".

With this statement one of my concerns was justified. A. Wiles did not prove (ELFLT). It is G. Faltings who did it. In this paper we will show that Faltings' (ELFLT) is equivalent to the original (EFLT) proved by Baica [2] using the Generalized Euclidean Algorithm (BGEA). (ELFLT) is equivalent to (not the same as) the original Fermat's Last Theorem (EFLT) stated by Fermat in Euclidean terms.

2 Historical survey of the development of the Generalized Euclidean Algorithm (BGEA)

The basis of the proof of (EFLT) is (BGEA) and we shall therefore give a short historical survey of its development. It all started with the very well known Euclidean Algorithm (EA).

Let the starting vector be $a^{(0)} = \left(a_1^{(0)}\right) \in \mathbf{R}$ and a transformation function, which is the greatest integer function $\left[a_1^{(0)}\right]$ as a companion vector $b^{(0)} = \left[a_1^{(0)}\right] = \left(b_1^{(0)}\right) \in \mathbf{R}$; then the recursive formula

$$a^{(v+1)} = \left(a_1^{(v)} - b_1^{(v)}\right)^{-1} \tag{2.1}$$

applied to the vector becomes a sequence $\{a^v\}, v = 0, 1, ...,$ which is called the continued fraction interpretation of the (EA). This formula (2.1) defines the continued fractions algorithm, which coincides with the (EA). L. Euler (1737) and L.J. Lagrange (1770) proved that every real quadratic irrational is represented by a periodic infinite simple continued fraction sequence and that every periodic infinite simple continued fraction sequence represents a real quadratic irrational. This theorem is known as Lagange's Theorem and for precision we call it Euler-Lagrange Theorem for quadratics (2-ELT).

(2-ELT) proves that every quadratic irrational makes (EA) periodic and it proves that the (EA) is always periodic. The unrestricted periodicity of the (EA) is a very strong property and because of this many important results in the algebraic number theory in quadratics were completely solved from its periodicity. The same problems remained open in n-dimensions, because there did not exist an unrestricted periodic Generalized Euclidean Algorithm (GEA) to solve them from its periodicity. Some previous results proved from the (EA) unrestricted periodicity are:

a) Construction with the ruler and the compass of the quadratic irrationals on the real line.

b) Every real quadratic irrational can be represented by an infinite periodic continued fractions (EA) development. This is (2-ELT).

c) Explicit solutions of the Euler - Pellian equation (EPE) $x^2 - ay^2 = \pm 1$ and ± 4.

d) The problem of finding the multiplicative group of units in the quadratic algebraic number fields was completely solved when (EPE) were completely solved. This is Dirichlet's problem for $n = 2$.

e) The existence of an algorithm to approximate quadratic irrationals.

f) The existence of the quadratic equation formula.

g) The existence of the parametric integer solutions of $x^2 + y^2 = z^2$.

h) To obtain the original Fibonacci numbers (2-DFN).

The corresponding questions for $n > 2$ are:

A) The proof of the one to one correspondence between the real numbers and the oriented straight line.

B) General simple continued fractions algorithm (GCFA) known as Hermite's problem.

B*) An n-dimensional equivalent of Euler-Lagrange theorem from quadratics (n-ELT) and this is to invent the generalized Euclidean algorithm (GEA) and to prove its unrestricted periodicity known as Hilbert's Universal Algorithm Periodicity problem (HUAPP) or Hilbert's "Zahlbericht".

C) Explicit algorithmic solutions for higher degree Diophantine equations.

D) Dirichlet's problem for any n.

E) The existence of a periodic algorithmic approximation of higher degree irrationals once the Hilbert completeness axiom was accepted.

F) To find relations between roots and coefficients for higher degree polynomials, as related to Galois' theory of polynomials.

G) The ability to prove the original Euclidean Fermat's Last Theorem (EFLT), that is to show that no positive integer solutions exist for $x^n + y^n = z^n$ if $n > 2$.

H)To obtain n-dimensional Fibonacci numbers (n-DFN).

All of these questions in n-dimension remained open and this caused D. Hilbert to ask for a universal algorithm as powerful as (EA) for $n = 2$ in order to solve all of the previously mentioned problems in higher dimensions from the periodicity of this universal algorithm. This Hilbert's "Zahlbericht" is known as (HUAPP) like B*. Logicians proved that Hilbert's hoped - for algorithm which would be unrestrictedly periodic and which would solve all of these above mentioned problems from its periodicity, does not exist. No explicit algorithmic solution of (HUAPP) was given.

Mathematicians had almost abandoned hope of obtaining further information about the arithmetic properties of Higher degree algebraic irrationals by means of simple continued fraction or (EA), when C.G.J. Jacobi [13] generalized the Euclidean algorithm for the cubic case (JA).

In 1839, C.H. Hermite [12] in one of his letters to Jacobi, challenged Jacobi to find an algorithm to develop irrationals of any degree into periodic sequences. Hermite was asking for the General Simple Periodic Continued Fractions Algorithm (GPCFA). But it was only after thirty years of frustration that Jacobi in 1868 extended (EA) methods to successfully represent some cubic irrationals by means of simple continued fractions. An application of the (JA) starts with the initial vector $a^{(0)} = \left(a_1^{(0)}, a_2^{(0)}\right) \in \mathbf{R}^2, n = 3$, the components of which are algebraic numbers. By use of the greatest integer function a "companion vector"

$$b^{(0)} = \left(b_1^{(0)}, b_2^{(0)}\right) \in \mathbf{R}^2, \text{ with } b_i^{(0)} = \left[a_i^{(0)}\right], i = 1, 2$$

is defined. A recursive transformation

$$a^{(v+1)} = \left(a_1^{(v)} - b_1^{(v)}\right)^{-1} \left(a_2^{(v)} - b_2^{(v)}, 1\right) \tag{2.2}$$

is constructed and applied to these vectors. Then the sequence $\{a^{(v)}\}$, $v = 0, 1, 2, ...$; is called Jacobi Algorithm (JA) of $a^{(0)}$.

The difficulties associated with this work are many. Jacobi's results were confined to a few examples in a cubic field. In spite of all Jacobi's efforts Hermite's problem remains unsolved.

In 1907, O. Perron [14] generalized the work of Jacobi. This generalization is known as the Jacobi-Perron Algorithm (JPA). In its general form, as defined by Jacobi for $n = 3$ and by Perron for $n \geq 2$, an application of the (JPA) starts with the definition of an initial vector

$$a^{(0)} = \left(a_1^{(0)}, a_2^{(0)}, ..., a_{n-1}^{(0)}\right) \in \mathbf{R}^{n-1}, n \geq 2,$$

the components of which are algebraic numbers. By use of the greatest integer function a "companion vector"

$$b^{(0)} = \left(b_1^{(0)}, b_2^{(0)}, ..., b_{n-1}^{(0)}\right) \in \mathbf{R}^{n-1}$$

with

$$b_i^{(0)} = \left[a_i^{(0)}\right], i = 1, 2, ..., n-1$$

is defined. A recursive transformation

$$a^{(v+1)} = \left(a_1^{(v)} - b_1^{(v)}\right)^{-1} \left(a_2^{(v)} - b_2^{(v)}, ..., a_{n-1}^{(v)} - b_{n-1}^{(v)}, 1\right) \tag{2.3}$$

is constructed and applied to these vectors. Then the sequence $\{a^{(v)}\}, v = 0, 1, 2, ...$, is called (JPA). Perron generalized Jacobi's methods to irrationals of any degree but since the choices of starting vector and transformation are difficult to make, he was also limited to a few periodic developments of higher degree irrationals. With all Perron efforts periodicity in proving Hermite's problem remains open. Perron was more successful in showing that if a development is periodic then the components of the initial vector are algebraic numbers. This result was general in the Lagrange direction of (n-ELT).

No further progress occurred on these problems until Helmut Hasse and Leon Bernstein [10] turned their attention to them. In 1965, Hasse and Bernstein started with an algebraic extension of the rational numbers, $Q(w)$, where w takes the form $w = \sqrt[n]{D^n + d}$ with $P(x) = \left(\prod_{i=1}^{n}(x - D_i^n) - d\right), d \in \mathbf{Z}, D_i \in \mathbf{N}$ and $d \mid D$.

$$a^{(0)} = ((w - D_1)(w - D_2)...(w - D_{n-1}), ..., (w - D_1)(w - D_2), (w - D_2))$$

with $b^{(0)} = a^{(0)}(D_1)$.

They showed that certain significant restrictions on D and d led to a (JPA) that was purely periodic.

1) For $d > 0$ they proved that the (JPA) of $a^{(0)}$ is purely periodic when $D \geq (n-2)d, d \mid D$ and $n \geq 3$,

and

2) For $d < 0$ the sequence is also purely periodic when $D \geq 2(n-1)d, d \mid D$ and $n \geq 3$.

With these conditions, the length of the period is $n(n-1)$. For this approach the periodicity remains an open problem since there are bounds on D and the restriction $d \mid D$ must hold. For example, no periodicity for $w = \sqrt[5]{12^5 + 6}$ can be proved under Hasse-Bernstein algorithm (HBA) restrictions since $12 \geq (5-2)6 = 18$ is not true. The (HBA) results were limited by their choices of w as real numbers. The shortcomings of this very important result are the restriction on d and the bounds on D. As of this result the periodicity of (HBA) is an open question, too.

In 1980, the author [1] defined a modification of the (JPA) that used the Hasse and Bernstein initial vector, but was not restricted to the real numbers. For the first time the complex numbers were considered. The only differences in the definitions stated alone are that the D_i's are now complex numbers. An immediate consequence of this extensions is that the bounds on D in the (HBA) are now eliminated and only the divisibility condition, $d \mid D$, remains. Returning to the example cited above, it can now be seen that $w = \sqrt[5]{12^5 + 6}$ has a periodic (ACF) development, only $6 \mid 12$ is required. At that time Baica proved only that $d \mid D$ is a necessary condition to make her algorithm to be periodic and named her algorithm, the Algorithm for Complex Field (ACF). Later, when Baica [2] proved that $d \mid D$ is also a sufficient condition and it can not be eliminated for the periodicity of her (ACF) algorithm, (ACF) became the General Euclidean Algorithm (GEA), and this algorithm is called Baica's Generalized Euclidean Algorithm (BGEA). (BGEA) is Euler direction and together with Perron result for Lagrange direction, Perron-Baica theorem (BPT) [9] becomes (n-ELT).

(BGEA) is the evolutionary development of the algorithms of Euclid, Jacobi, Perron, Hasse, Bernstein and Baica. In many other published papers the author has extended the applications of the restricted periodicity of (BGEA) to produce solutions for all open n-dimensional problems listed at the beginning of this paper and we do not exaggerate when we say that (BGEA) produces solutions of almost all complicated unsolved problems in algebraic number theory. What it is more important is that we proved [8] that (BGEA) is the Euler System (ES) for the Algebraic Number Theory in n dimensions exactly as (EA) is the (ES) in quadratics.

As we see, it is not only the beginning (EA) and the end (BGEA), but there is so much else in the gap that separates them by more than 2000 years. (BGEA) puts together the work of great mathematicians during the entire history of mathematics beginning with Euclid and finishing with Baica. All of these great mathematicians aimed at producing the (GEA) and to prove its periodicity sometime in their life, and with their seminal work which the author put together, their dreams became a reality and now we have (BGEA) to be the only Generalized Euclidean Algorithm (GEA).

I have the greatest respect for the inspiring work of these great mathematicians before me, who historically paved my way to finish this final step and give mathematics this very powerful tool (BGEA) which is the Generalized Euclidean Algorithm, the Euler System (ES) of the algebraic number theory. Since (BGEA) is of the same cut or prototype as (EA) where $n = 2$ in the dimension n of (BGEA) became (EA), then (BGEA) is the only possible Generalized Euclidean Algorithm (GEA).

3 The Euclidean character of Fermat's Last Theorem (EFLT) and its proof in Euclidean

In [3] the author expressed her genuine concern about the proof of Fermat's Last Theorem in the geometry of the elliptic curve (ELFLT), which may not be equivalent to (let alone the same as) the result in the Euclidean geometry where Fermat's Last Theorem originated (EFLT) about three hundred fifty years ago.

In 1940, E. Schmidt was the first to say that, in mathematics, a problem can be approached geometrically, algebraically or analytically and proofs can be given in different mathematical varieties, which are defined in modern mathematics as mathematical models. Also, he said that since we cannot compute in a geometry, we have to construct an algebra for that corresponding geometry. He is the father of the General Algebraic Geometry as K.F. Gauss is the father of the Algebraic Number Theory, which is the algebra of the n-dimensional Euclidean Geometry (E^nG). E. Schmidt introduced for the first time the algebra of the elliptic curves and elliptic functions and Germans continued from there. E. Schmidt was the doctoral father of J. Schmidt who, together with H. Hasse were the doctoral fathers of the author at the University of Houston, where she earned her PhD in Mathematics (Algebraic Number Theory and Universal Algebra) in August 1980. Regretfully both died at the end of the year after the author finished her PhD (Jürgen Schmidt on October 26, 1980 and Helmut Hasse on December 24, 1980).

In mathematics we can construct as many geometries or geometrical models as we please. All that we need is to have the elements declared, to state the axioms and the definitions and to have consistency in our mathematical logic.

All of these many geometries do not relate to each other, but they all relate to the topology. Because of this reason, if we prove something in one geometry it may not be the same as in another geometry. Only one geometry is the Euclidean Geometry (EG), the other geometries are non-Euclidean geometries. For example if we consider V-th postulate in (EG) when two parallel lines do not intersect, they intersect at two ideal points Ω and Ω' in the Hyperbolic Geometry (HYG).

For the (EG) the elements are points and the lines are the straight lines, and for the geometry of elliptic curves (GELC) the elements are points and the lines are the elliptic curves. We have to make distinction between what is an element in a geometry and what is a definition in a geometry. In the (GELC) the elliptic curves are elements while in the (EG) the elliptic curves are definitions. The algebra corresponding to a geometry is called the "Number Theory" or the "Arithmetic" of that geometry. We know that *no "Geometry of the Elliptic Curves" (GELC) and no "Number Theory or Arithmetic of the Elliptic Curves" known as Hecke and Langrands Algebra (HLA), which is the number theory of the (GELC), existed 350 years ago, and we are forced to recognize the strong Euclidean character of the Original Fermat's Last Theorem (EFLT).* Therefore we have to follow Hasse's advice to solve the original (EFLT) in the Algebraic Number Theory, which is the algebra of the n-Dimensional Euclidean Geometry (E^nG).

Remark 1 *In some of her papers about the proof of (EFLT) the author named Hilbert's "Zahlbericht" to find an universal always periodic algorithm (HUAPP) as Hilbert's 10-th problem, and since some mathematicians seem to object to this nomenclature we can drop "10-th problem" and call it as in her first paper [2], this "Hilbert's (dreamed) universal always periodic algorithm periodicity problem (HUAPP)".*

Theorem 2 ***(EFLT).*** *(BGEA) restricted periodic when $n \geq 3$ implies that there are no integer solutions for $x^n + y^n = z^n$, if $n \geq 3$.*

Proof. *It is known that in quadratics $x^2 + y^2 = z^2$ has integer solutions given in a parametric form and this is an immediate consequence of the fact that (EA) is always periodic by (2-ELT). We know that (EA), (JA), (PA), (JPA) and (HBA) developed only real numbers and (HBA) was the closest algorithm over the reals to the (BGEA). (BGEA) was developed for the first time over the complex numbers and proved its restricted periodicity for $w = \sqrt[n]{D^n + d}$ where only the restriction $d \mid D$ remains and since no other larger set of numbers in range exists having the complex numbers for a subset as $\mathbf{R} \subset \mathbf{C}$ in Euclidean this shows that (BGEA) is the only (GEA). Thus, we proved that (BGEA) is the only explicit form of Hilbert's demanded universal algorithm, from whose always periodicity all of the open problems in n-dimensions could be solved. The same have been already solved in quadratics ($n = 2$) from the always periodicity of (EA) under the form of continued fractions. Logicians proved that Hilbert's hoped for periodic algorithm does not exist and this is known as (HUAPP). We proved exactly the same result, providing mathematics with an explicit solution* *for (HUAPP) where (BGEA) is periodic for any higher degree algebraic number* $w = \sqrt[n]{D^n + d}$ *if* $d \mid D$. *Putting those two together it is true that if* $d \nmid D$, *(BGEA) is not periodic, since otherwise it will contradict (HUAPP). Hilbert proved that* $x^2 + y^2 = z^2$ *has integer solutions because (EA) is periodic using the solvability by radicals. That is: the dimension of the (EA) is* $n = 2$ *since (2-ELT), and the degree* $n = 2$ *of* $x^2 + y^2 = z^2$ *is related with the dimension of the (EA) where this is related with the multiplicative group of units in the corresponding fields (Dirichlet's problem for* $n = 2$*). Therefore, if* $n \geq 3$ *there does not exist positive integers* x, y, z *such that* $x^n + y^n = z^n$ *since (BGEA) is not always periodic for* $n \geq 3$, *and there exists positive integers* x, y, z *such that* $x^2 + y^2 = z^2$ *since for* $n = 2$ *(BGEA) for reals becomes (EA) and (EA) is periodic for any quadratic irrational. If (BGEA) would be periodic for any* n, *then* $x^n + y^n = z^n$ *would have integer solutions but this will contradict (HUAPP). Therefore (BGEA) not periodic for* $n \geq 3$ *if* $d \nmid D$ *implies (EFLT). The degree* n *of the equation is related with Dirichlet's problem, which the author proved that this is solved only if* $d \mid D$ *in (BGEA) and this is related with the dimension* n *of (BGEA), which is given by the degree* n *of the irrational, which makes (BGEA) restrictively periodic (n-ELT).* ■

In conclusion (BGEA) is a very powerful algorithm when it becomes periodic (ES). The proof of (EFLT) is the conclusion of the results in all author's papers over the years. The (BGEA) will have the same importance in mathematics for higher dimensional fields in the years to come, exactly as (EA), $n = 2$ in (BGEA), for mathematics in quadratic fields more than 2000 years in the past.

Conclusion 3 *This problem (EFLT) has baffled the best mathematicians for nearly 350 years. We, finally, proved that the restricted periodicity of (BGEA) for* $n \geq 3$ *implies (EFLT). Fermat himself intended to prove (EFLT) by induction since the classical number theory is a Peano algebra and there the induction never fails to give the generalization. He first proved his conjecture to be true for* $n = 4$ *but when he wanted to do the induction on* n *he could not use the degree of the equation. The induction must be done on the dimension* n *of the (BGEA) where* $n = 2$ *in (BGEA) is (EA), and the dimension is given by the degree of the irrational which makes (BGEA) restrictive periodic. The dimension of (BGEA) brings us to the degree of the equation from Dirichlet's problem to the solvability by radicals. At that time Fermat did not have (BGEA) to perform induction on its dimension. The equation* $x^n + y^n = z^n, n \geq 3$ *does not resemble any of the Diophantine equations solved by the author in [8] (Dirichlet's problem for* $n \geq 3$*) using units in the algebraic number fields, by the restricted periodicity of (BGEA), and this is the reason why (BGEA) restricted periodicity is equivalent to (EFLT).*

4 A. Wiles' attempt to prove the elliptical Fermat's Last Theorem (ELFLT)

We are not going to reproduce A. Wiles's work here, our intention is to underline the weakness of his proof in his attempt to prove (ELFLT).

Modularity is essential in the proof of (ELFLT). Using deformations, the constructed l-adic representation for $l = 3$, starting with the representation on the 3-division points, known to be congruent to a modular representation, leads to the following commutative diagram:

$$\begin{array}{ccccc} & & \widehat{T} & \longrightarrow & \frac{T}{m} \\ & \nearrow & & & \\ R & & & & \uparrow \\ & \searrow & & & \\ & & Z_3 & \longrightarrow & F_3 \end{array}$$

A. Wiles wanted to show that R is isomorphic to $\widehat{T}$ since then the elliptic Galois representation becomes modular. There is no information about R from the general principles of its construction explained in the proof. With special considerations in the proof $R = \widehat{T}$ if and only if $order\frac{p}{p^2} = order\frac{Z_3}{\eta Z_3}$. Let us denote the order by $|...|$.

(4.1) $$\left|\frac{p}{p^2}\right| = \left|\frac{Z_3}{\eta Z_3}\right|$$

A. Wiles tried to establish equality by using Euler systems given by Kolyvagin. The equality could not be proved because Euler systems of higher levels could not be constructed. Thus, A. Wiles did not prove (ELFLT).

5 G. Faltings finishes the proof of (ELFLT)

The July 1995, *Notices of the AMS* published a translation from the German of G. Faltings' March 1995 article [11] in which the author said at the beginning of the paper: "The proof of the conjecture mentioned in the title was finally completed in September 1994. A. Wiles announced this result in the summer of 1993. However, there was a gap in his work. The paper of Taylor and Wiles does not close this gap but it circumvents it. This article is an adaptation of several talks that I have given on this topic and is by no means about my own work. The specialists can then alleviate their boredom by finding these mistakes and correcting them."

With this statement one of my concerns was justified. A. Wiles did not prove (ELFLT). It is G. Faltings who did it. I am specialist in Algebraic Number Theory (regrettably very few left in this field) but my common sense as a mathematician tells me to accept G. Faltings' proof of (ELFLT).

Faltings began where A. Wiles got stuck. He starts from the minimal level M and then reduces to it. The R_e and $\widehat{T}_e$ are rings of power series and they become equal at the limit. R is obtained from R_e and $\widehat{T}$ from $\widehat{T}_e$. To prove the equality (4.1) is to reduce the problem to the minimal case and he estimates how both sides of the inequality

$$\left|\frac{p}{p^2}\right| \geq \left|\frac{Z_3}{\eta Z_3}\right| \tag{5.1}$$

changes as one proceeds from minimal level M to a higher level N. An upper bound for the left hand side and a lower bound for the right hand side were found, and they coincided. This ends the proof of (ELFLT).

6 Elliptical Fermat's Last Theorem (ELFLT) is equivalent to (but not the same as) the original Fermat's Last Theorem (EFLT) stated by Fermat in Euclidean terms

As we have said before among the many geometries constructed only one is the Euclidean Geometry. All of the others are non-Euclidean. The geometries do not relate to each other, but they all relate to the topology. Because of this we can prove something in one geometry which can not be proved in another geometry, or it may mean something else in another geometry. The best that we can get is that these results are equivalent or isomorphic. Also, these results should be transferred in the computing algebras of the corresponding geometries. To prove that two results are equivalent in two different computer algebras we have to provide the transformation, which is one to one correspondence function. To prove that the results are the same in these two different categories (computing algebras) we need the Galois' connection, which requires that this transformation to be analytical continuous. In other words we need to provide a one to one, onto, analytical continuous function, which is a functor. When we transfer a result from one computing algebra system to be the same in another computing algebra system we need this functor. So far in Faltings' proof of (ELFLT) only the transformation for the equivalence was given since both proofs Baica's of (EFLT) and Faltings' of (ELFLT) bring us to the solvability by radicals, and this is related with the multiplicative group of units in the corresponding fields (Dirichlet's problem) in Euclidean.

In order to provide the functor in Elliptic they have to prove that the equation $x^2 + y^2 = z^2$ has the same parametric solutions like in Euclidean $x = u^2 - v^2, y = 2uv, z = u^2 + v^2$, with $\gcd(u, v) = 1$ for primitive solutions. This is impossible since in elliptic it starts with $n = 3$.

7 Important conclusion remark

G. Faltings proved (ELFLT). Baica proved (EFLT). Also, she proved that (ELFLT) *is equivalent to* (EFLT) *and not the same* since we can not provide the functor.

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Author's address
The University of Wisconsin
Department of Mathematical & Computer Sciences
53190, Whitewater, Wisconsin, U.S.A.
baicam@uww.edu